Are spacetime horizons higher dimensional sources of energy fields? (The black hole case).

Manasse R. Mbonye

Michigan Center for Theoretical Physics
Physics Department, University of Michigan, Ann Arbor, MI 48109

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Abstract

We explore the possibility that spacetime horizons in 4D general relativity can be treated as manifestations of higher dimensions that induce fields on our 4D spacetime. In this paper we discuss the black hole event horizon, as an example (we leave the cosmological case for future discussion). Starting off from the field equations of gravity in 5D and some conditions on the metric we construct a spacetime whose imbedding is a 4D generalization of the Schwarzchild metric. The external region of the imbedded spacetime is found to contain two distinct fields. We discuss the properties of the fields and the potential implications. Taken as they are, the results suggest that the collapse of matter to form a horizon may have non-local consequences on the geomerty of spacetime. In general, the use of horizon-confined mass as a coordinate suggests three potential features of our universe. The first is that the observed 4D spacetime curvature and ordinary matter fields can be identified as hybrid features of 5D originating from the mixing of coordinates. Secondly, because the fifth coordinate induces physical fields on the 4D hyperface, the global metric of the universe can not be asymptotically flat. And finally, associating matter with an independent dimension points towards a theory of nature that is scale invariant.

1 Introduction

In an effort to build a unified theory of nature to explain the observed universe, physics has had to look up to higher dimensions. A consequence of this effort has been the development, in recent years, of higher dimensional models, notably the Superstring models[1] and M-Theory[2]. In these models dimensions are usually assumed to be curled up into length scales of order of Planck size, hence rendering them difficult to observe at any energies lower than the Planck energy. This modern view of higher dimensional compactification has its historical roots in Klein's original interpretation[3] of (the stronger) Kaluza's *cylinder condition*[4] (which we discuss later). Lately, alternatives to compactification such as the Randall-Sundrum mechanism[5] have been proposed.

Dimensional Compactification is certainly consistent with why we don't sense the higher dimensions. Whether such a feature is also necessary or whether nature may have other options is still academic. It is however reasonable and may be prudent, at this time, to explore further alternative possibilities. In this respect, one is reminded of the citizens of the legendary 2D world in *Flatland*[6] and their limitations in perception. These citizens' understanding of concepts like angular momentum, or the force on an electric charge moving in a magnetic field, requires them to develop a higher dimensional theory involving cross-products. Clearly, the 'other' dimensions that form part of the manifold in which Flatland is imbedded do not have to be compact. Flatlanders are just two dimensional beings living in their two dimensional world with their observations in the third dimension restricted by what they have learned to call a horizon. As a result, Flatlanders would interpret effects of their interactions with the higher dimensional world as originating from the horizon.

In this discussion we seek for possible manifestations and implications of higher dimensions based on the assumption that such dimensions are not necessarily compact. In our approach we represent matter as the fifth coordinate, provided such matter is confined in a spacetime horizon. The the fifth coordinate is then given by the length $x^4 = h$ associated with the horizon size. The criterion employed in this work to treat a quantity as coordinate representing a dimension, is the quantity's independence from the other spacetime dimensions. In so far as a spacetime horizon (local or global) signifies a boundary to our observable 4D universe, then the length h associated with the horizon size meets the above criterion and its treatment as a manifestation of a dimension can be justified on this basis.

The possibility of representing matter as a coordinate has been suggested before by Lessner[7] and by Wesson[8]. It is however difficult to justify the concept of a fifth dimension without the above criterion, namely the independence of the associated coordinate. From the onset it is clear, for example, that in our approach ordinary matter cannot be justifiably treated as an independent coordinate. This is because the worldline of any such matter (including particles) is associated with a 4D spacetime measure and is therefore not independent of the spacetime coordinates. Such an identification distinguishes our treatment, its results and implications, from the previous treatments. As we point out in the next section, ordinary matter (i.e. not trapped inside an event horizon) is seen to be a hybrid, resulting from the mixing of spacetime coordinates with the fifth coordinate.

We shall demonstrate that given a 5D spacetime geometry in which the fifth coordinate x^4 is treated as a length h corresponding to the horizon size, then there is an imbedded 4D spacetime on which fields are induced. In particular, taking the horizon size as that of a black hole mass M so that $h = 2GMc^2$ we find that, in the case of the induced 4D geometry, the external space outside the black hole is filled with two distinct fluids. We discuss the properties of these fields and their implications.

2 The 5D field equations

2.1 The fifth dimension

We begin by introducing the physical basis for the fifth coordinate. One can build an intuitive sense of the fifth dimension by drawing an analogy with the time dimension[9]. In the Minkowiski spacetime, the time dimension is associated with a length coordinate written as $x^0 = ict$, where c is a universal constant (the upperbound speed for propagation of physical information) . Because of the large value of this constant, we do not usually sense time as a dimension except at high velocities (comparable to c) where such relativistic phenomena as length contraction and time dilation manifest themselves.

Analogously, in our present approach which is geometric, one can associate the fifth dimension with a coordinate

$$x^4 = 2\sqrt{\varepsilon}\varkappa M = \sqrt{\varepsilon}h, \tag{1}$$

constructed from a mass M confined in an event horizon of size h. Here, the parameter $\varepsilon = \pm 1$ identifies the coordinate as space-like or time-like and $\varkappa = Gc^2 \approx l_p m_p$ is a universal constant with units of length per unit mass and gives the length scale physically associated with maximum untrapped mass. In MKS units $\varkappa = 7.42 \times 10^{-28} mkg^{-1}$. Because of the small value of $\varkappa$, the observable effects of the fifth dimension (analogous to time dilation/length contraction) should be favored by high density fields. One notes, however, that $\varkappa$ is independent of velocity. Thus, even at ordinary densities (e.g. earth density) effects of higher dimensions should be readily observable in our universe, as deviations from Minkowiski spacetime, provided a large enough interval associated with the fields is considered. With regard to the specific nature of the (trapped-mass)-spacetime coordinate mixing effects, one expects a bending of spacetime analogous to the special relativistic length contraction and a 'dilution' effect of the mass coordinate corresponding to the time dilation. Such effects would give rise to spacetime curvature and introduce matter fields in 4D. The suggestive implication, then, is that ordinary 4D spacetime curvature and ordinary matter/radiation are manifestations of higher dimensions.

That curvature is imposed on the 4D spacetime is a known feature of general relativity. The only addition our treatment suggests is that such imposition results from the mixing of the fifth coordinate with the spacetime ones. On the other hand, the observation that ordinary matter fields in 4D also originate from such coordinate mixing is a broad concept to be a subject of this initial and future discussions. We will comment on the cosmological and other implications of these effects later in the conclusion of this discussion.

2.2 The equations

To proceed, we first a give a summary of the equations which set the framework for the remaining discussion. These equations can be derived from the five dimensional form of the Einstein action

$$S = 116\pi\bar{G}\int\sqrt{-g}Rd^5x \tag{2}$$

by varying the former with respect to the 5D metric g_{ab}. Here R and $\bar{G}$ are the 5D Ricci scalar and Newton's constant, respectively and g is the determinant of the full 5D metric whose line element is

$$dS^2 = g_{ab}dx^a dx^b,\ (a, b = 0, .., 4)\,. \tag{3}$$

In our notation, lower case lettering is used for spacetime indices (Roman for 5D and Greek in 4D) and from now on we stick to the geometrized units $8\pi G = c = 1$, unless otherwise stated.

As already mentioned above, our discussion is based on the assumption that spacetime horizons can be treated as manifestations of higher dimensions. With this assumption we investigate the possibility that such horizons are generators of matter fields on the 4D spacetime. One does expect that the field equations derived from Eq. (2) should give a 5D geometry g_{ab} whose foliation yields a family of 4-hyperfaces with physically meaningful interpretation. In particular, we suppose in this treatment that the metric induced on the 4D hyperface describes the spacetime outside a *static* source which is represented by the horizon size. This puts constraints on the properties of such a metric induced on the 4D manifold, and hence on the foliation character of the 5D space. Thus we expect that the resulting 4D spacetime should:

- (i) have spherical symmetry;
- (ii) reduce to the Schwarzschild solution[10] on a constant h surface;
- (iii) be asymptotically flat (as $h \longrightarrow 0$).

In passing, one notes with regard to condition (iii), that since in a cosmological sense h never realistically goes to zero then the geometry of the universe could never be globally Minkowski. This suggests, independently, that the 4D universe should, necessarily, be bathed in some vacuum, Λ, with an associated non-vanishing energy density ρ_Λ. We will revisit this issue in a future discussion of the cosmological case.

Condition (i) demands that the geometry of the full 5D manifold should admit a metric of the form

$$dS^2 = e^{\nu}dt^2 - e^{\mu}dr^2 - R^2 d\Omega^2 + \varepsilon e^{\psi}dh^2, \tag{4}$$

where, condition (ii) implies that the metric coefficients can, at most, be functions of r and h only.

Before proceeding, it is worthwhile to comment on this dependence of the induced 4-metric $g_{\mu\nu}$ on the fifth coordinate, h. In his original work, Kaluza[4] assumed that geometric and physical objects in 4D are independent of the fifth coordinate. This assumption is expressed in the *cylinder condition* which holds that all derivatives of the 4D metric with respect to the fifth coordinate, must vanish, i.e. $\partial_{(a=4)}\left[g_{\mu\nu}\right] = 0$. Later, Klein[3] introduced a weaker condition by letting the full 4-metric $g_{\mu\nu}\left(x^{\mu}, x^{4}\right) = g^{0}_{\mu\nu}\left(x^{\mu}\right) X\left(x^{4}\right)$, be separable in the variables x^{μ} and a compactified x^4. This view has been introduced in the modern string theories. It holds that the higher dimensions are curled up into a length scale of order of Planck size, hence rendering them difficult to observe at any energies lower than the Planck energy. Throughout our treatment the *cylinder condition* is relaxed. As we shall find it is the relaxation of this condition which, in our approach, facilitates the introduction of matter fields into the $4D$ spacetime from higher dimensions.

With the form of the above line element one finds that the only surviving components of the $5D$ Ricci tensor, R_{ab}, are:

$$\begin{aligned}
R_{00} &= e^{\nu-\mu}\left[\nu''2 + \nu'^2 4 - \nu'\mu'4 + \nu'\psi'4 + \nu'R'R\right] + \varepsilon e^{\nu-\psi}\left[-\overset{\diamond\diamond}{\nu}2 - \overset{\diamond}{\nu}^2 4 - \overset{\diamond}{\nu}\overset{\diamond}{\mu}4 + \overset{\diamond}{\nu}\overset{\diamond}{\psi}4 - \overset{\diamond}{\nu}\overset{\diamond}{R}R\right]; \\
R_{11} &= -\nu''2 - \psi''2 - \nu'^2 4 - \psi'^2 4 + \nu'\mu'4 + \nu'\psi'4 + \mu'R'R - \varepsilon e^{\mu-\psi}\left[-\overset{\diamond\diamond}{\mu}2 - \overset{\diamond}{\mu}^2 4 + \overset{\diamond}{\nu}\overset{\diamond}{\mu}4 + \overset{\diamond}{\nu}\overset{\diamond}{\psi}4 - \overset{\diamond}{\nu}\overset{\diamond}{R}R\right]; \\
R_{22} &= 1 - R^2 e^{-\mu}\left[\left(R'R\right)^2 + R''R + R'2R(\nu' - \mu' + \psi')\right] + \varepsilon R^2 e^{-\mu}\left[\left(\overset{\diamond}{R}R\right)^2 + \overset{\diamond\diamond}{R}R + \overset{\diamond}{R}2R(\overset{\diamond}{\nu} - \overset{\diamond}{\mu} + \overset{\diamond}{\psi})\right]; \\
R_{33} &= \sin^2\theta\left(R_{22}\right); 5 \qquad (1 \\
R_{44} &= -\overset{\diamond\diamond}{\nu}2 - \overset{\diamond}{\nu}^2 4 - \overset{\diamond\diamond}{\mu}4 - \overset{\diamond}{\mu}4^2 + \overset{\diamond}{\nu}\overset{\diamond}{\psi}4 + \overset{\diamond}{\nu}\overset{\diamond}{\psi}4 - \overset{\diamond}{\psi}\overset{\diamond}{R}R - \overset{\diamond\diamond}{R}2R + \varepsilon e^{\psi-\mu}\left[\psi''2 + \psi'^2 4 + \nu'\psi'4 + \mu'\psi'4 + \psi'R'R\right] \\
R_{14} &= -\overset{\diamond}{\nu}'2 - \nu\overset{\diamond}{\nu}'4 + \overset{\diamond}{\mu}\nu'4 + \overset{\diamond}{\nu}\psi'4 + \overset{\diamond}{\mu}R'R + \psi'\overset{\diamond}{R}R - 2\overset{\diamond}{R}'R.
\end{aligned}$$

Here the over head diamond '$\diamond$' means differentiation with respect to the fifth coordinate while the prime '$'$', as usual, denotes differentiation with respect to the radial coordinate.

Now, conditions (i) and (ii) suggest that, as an ansatz to the above equations, we take a $5D$ line element of the form

$$dS^2 = (1-hr)\,dt^2 - (1-hr)^{-1}\,dr^2 - r^2 d\Omega^2 - \varepsilon\phi dh^2, \tag{6}$$

where $\phi = \phi(r,h)$ is a lapse function and $\varepsilon = \pm 1$, as mentioned before, identifies the h coordinate as either spacelike or timelike. In this particular work we assume a coordinate system in which ϕ is scaled to unity.

Using Eqs. (5) and (6) we find the only surviving components of the 5D Ricci tensor R_{ab} are

$$\begin{aligned} R_{00} &= -12\varepsilon r^2(1-hr); \\ R_{11} &= -12\varepsilon r^2(1-hr)^3; \\ R_{14} &= -12r^2(1-hr);7 \\ R_{44} &= 12r^2(1-hr)^2. \end{aligned} \tag{2}$$

Further, the $5D$ Ricci scalar is given as

$$R = R^a_a = 12r^2\varepsilon(1-hr)^2 \tag{8}$$

Using Eqs. (7) and (8) above to build the $5D$ Einstein tensor $G_{ab} = R_{ab} - 12Rg_{ab}$ we find its surviving components to be

$$\begin{aligned} G_{00} &= -34\varepsilon r^2(1-hr); \\ G_{11} &= -14\varepsilon r^2(1-hr)^3; \\ G_{14} &= -12r^2(1-hr); \\ G_{22} &= r^2 4\varepsilon r^2(1-hr)^2;9 \\ G_{33} &= \sin^2\theta G_{22}; \\ G_{44} &= 14r^2(1-hr) \end{aligned} \tag{3}$$

3 The 4+1 splitting and the induced fields

In order to isolate physical information from the above full 5D results it is worthwhile comparing such results with those from a $4+1$ splitting of the Kaluza-Klein theory. This comparison will manifest features in the preceding results which suggest the existence of two distinct fields induced on the 4D hyperface. To this end we start with an overview of a Kaluza-Klein theory with the *cylinder condition* relaxed, i.e. $\partial_{(a=4)}[g_{\mu\nu}] \neq 0$. In such an approach one can, in general, institute a $4+1$ split of the $5D$ metric $dS^2 = g_{ab}dx^a dx^b$. This foliation leaves an induced 4D metric, $g_{\mu\nu}(x^a)$, $(\mu,\nu = 0,..,3)$ which can be related to the $5D$ metric by[11]

, (10)